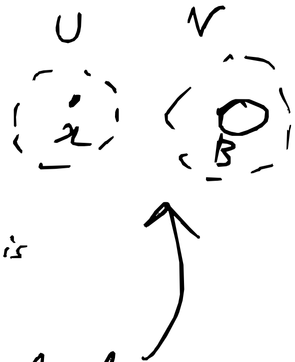
4/7

Urysohn's Lemma & Applications

Revisiting separability axioms:

We previously defined T_0, T_1 and T_2 spaces.

We have other classifications as well:

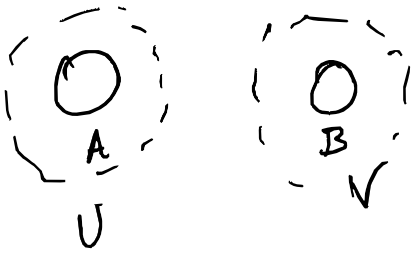


Let X be a space where each singleton set $\{x\}$ is closed.

T_3 (regular): For each pair of a point x and closed set B disjoint from $\{x\}$, $\exists$ open sets $U \ni x$ and $V \supseteq B$ s.t. $U \cap V = \phi$.

T_4 (normal): For each pair of disjoint closed sets A and B, $\exists$ open sets $U \supseteq A$ and $V \supseteq B$ s.t.

$$U \cap V = \phi.$$



Normal $\Rightarrow$ Regular.

Urysohn's Lemma: Let X be a normal space and A, B be disjoint non-empty closed sets in X. Then there exists a continuous function $f : X \longrightarrow \mathbb{R}$ such that

$$f|_A \equiv 0 \quad (\forall x \in A,\ f(x) = 0)$$

$$f|_B \equiv 1 \quad (\forall x \in B,\ f(x) = 1)$$